ERRATUM TO THE LOWER ALGEBRAIC K-THEORY OF Γ_3

IVONNE J. ORTIZ

Abstract. The lower algebraic K-theory of Γ_3 presented in the Main Theorem of [Or04] is incomplete. In this erratum we present the correct version of this theorem.

In [Or04] we prove the following result which turns out to be incomplete (see explanations below).

Main Theorem (Erroneous)**.** *Let* $\Gamma_3 = O^+(3,1) \cap GL(4,\mathbb{Z})$. *Then the lower algebraic K-theory of the integral group ring of* Γ_3 *is given as follows:*

$$\begin{aligned} Wh(\Gamma_3) &= 0, \\ \tilde{K}_0(\mathbb{Z}\Gamma_3) &\cong \mathbb{Z}/4 \oplus \mathbb{Z}/4, \\ K_{-1}(\mathbb{Z}\Gamma_3) &\cong \mathbb{Z} \oplus \mathbb{Z}, \ \textit{and} \\ K_n(\mathbb{Z}\Gamma_3) &= 0, \ \textit{for } n < -1. \end{aligned}$$

We accomplished these computations using the fact that Γ_3 satisfies the Farrell-Jones Isomorphism Conjecture in lower algebraic K-theory [FJ93]. One of the steps in this project was to classify up to isomorphism the family $\mathcal{VC}$ of all virtually cyclic subgroups of Γ_3 (see [Or04, Section 3]), and it is right here where the problem arises. The list of groups appearing in [Or04, Lemma 3.4] is *incomplete*.

Let us recall that any virtually infinite cyclic group is either of type $F \rtimes_\alpha \mathbb{Z}$, where F is a finite group, or it maps onto D_∞ with finite kernel (see [Or04, Lemma 3.2]). It is well known that the group $F \rtimes_\alpha \mathbb{Z}$ can occur in Γ_3 only if $C_{\Gamma_3}(F)$ contains an element of *infinite order* (see [Or04, pg. 340]).

In [Or04, pg. 341], we claim that $C_{\Gamma_3}(\langle S_1, S_2 \rangle) \cong C_{\Gamma_3}(D_2) = \langle\, S_4 \,\rangle \cong \mathbb{Z}/2$, $C_{\Gamma_3}(\langle S_1, S_3 \rangle) \cong C_{\Gamma_3}(D_3) = \langle\, S_1, S_3, S_4S_3S_4 \,\rangle \cong (\mathbb{Z}/2)^3$, but these computation are *erroneous*, (here D_n denotes the dihedral group of order $2n$). R. Akhtar pointed out to the author that in fact these groups are *infinite*. The following are the correct computations of these groups (see [N05]):

$$C_{\Gamma_3}(\langle S_1, S_2 \rangle) \cong D_\infty \rtimes \mathbb{Z}/2, \text{ and } C_{\Gamma_3}(\langle S_1, S_3 \rangle) \cong D_\infty \rtimes D_2.$$

This fact *alters* the classification of the virtually infinite cyclic subgroups of Γ_3 given in [Or04, Lemma 3.4]. This problem is *settled* in [LO, Section 3], where we classify up to conjugacy the *maximal virtually infinite cyclic* subgroups of Γ_3.

Appealing to this new information, we want to *emphasize* that the group $D_2 \times \mathbb{Z}$ is now part of the family $\mathcal{VC}$ of virtually cyclic subgroups of Γ_3, and that the **essential** problem arises here. It is well known that if $Q = D_2 \times \mathbb{Z}$, the relative assembly map appearing in [Or04, Theorem 2.2, and Theorem 4.7]) is *not* an isomorphism. Since $K_0(\mathbb{Z}[Q])$ is *infinitely generated* (see [F87], [BM67]), the map $\mathcal{A}_{\mathcal{F}_Q}$ appearing in [Or04, Theorem 4.7]) *fails* to be an isomorphism for $Q = D_2 \times \mathbb{Z}$. Therefore the reduction to finite subgroups *falls flat*.

In order to complete the computations of the lower algebraic K-groups of the integral group ring of Γ_3, we *must* compute the homotopy groups

$$H_n^{\Gamma_3}(E_{\mathcal{VC}}(\Gamma_3); \mathbb{KZ}^{-\infty}), \quad \text{for } n < 2.$$

These computations are carried out in [LO, Section 4, and Section 5]. The result of these computations can be summarized in the following theorem (see [LO, Theorem 1.1]):

Theorem 0.1. *Let $\Gamma_3 = O^+(3,1) \cap GL(4, \mathbb{Z})$. Then the lower algebraic K-theory of the integral group ring of Γ_3 is given as follows:*

$$\begin{aligned} Wh(\Gamma_3) &\cong \bigoplus_{\infty} \mathbb{Z}/2 \\ \tilde{K}_0(\mathbb{Z}\Gamma_3) &\cong \mathbb{Z}/4 \oplus \mathbb{Z}/4 \oplus \bigoplus_{\infty} \mathbb{Z}/2 \\ K_{-1}(\mathbb{Z}\Gamma_3) &\cong \mathbb{Z} \oplus \mathbb{Z}, \ \text{and} \\ K_n(\mathbb{Z}\Gamma_3) &\cong 0, \ \text{for } n < -1. \end{aligned}$$

where the expression $\bigoplus_{\infty} \mathbb{Z}/2$ refers to a countable infinite sum of $\mathbb{Z}/2$.

References

[BM67] H. Bass, and M. P. Murthy, *Grothendieck groups and Picard groups of abelian group rings*, Ann. Math. textbf86 (1967), 16-73.

[F87] F. T. Farrell, *A remark on K_0 of crystallographic groups*, Topology Appl.} **26** (1987), 97-99.

[FJ93] F. T. Farrell, and L. Jones, *Isomorphism conjectures in algebraic K-theory*, J. Amer. Math. Soc. **6** (1993), no. 2, 249-297.

[N05] K. Nuida, *On centralizers of parabolic subgroups in Coxeter groups*, arXiv:math.GR/0501061

[LO] J. F. Lafont, and I. J. Ortiz, *Relative hyperbolicity, classifying spaces, and lower algebraic K-theory*. Preprint

[Or04] I. J. Ortiz, *The lower algebraic K-theory of Γ_3*, K-theory **32** (2004), no. 4, 331–355.

Department of Mathematics and Statistics, Miami University, Oxford, OH 45056
E-mail address: ortizi@muohio.edu