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ERRATUM

The lower algebraic K-theory of Γ_3

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The lower algebraic K-theory of Γ_3 presented in the Main Theorem of [6] is incomplete. In this erratum we present the correct version of this theorem.

In [6] we stated the following result which turns out to be incomplete (see explanations below).

Main Theorem (Erroneous) *Let* $\Gamma_3 = O^+(3,1) \cap GL(4,\mathbb{Z})$. *Then the lower algebraic K-theory of the integral group ring of* Γ_3 *is given as follows*:

$$\begin{aligned} Wh(\Gamma_3) &= 0, \\ \tilde{K}_0(\mathbb{Z}\Gamma_3) &\cong \mathbb{Z}/4 \oplus \mathbb{Z}/4, \\ K_{-1}(\mathbb{Z}\Gamma_3) &\cong \mathbb{Z} \oplus \mathbb{Z}, \ \textit{and} \\ K_n(\mathbb{Z}\Gamma_3) &= 0 \ \textit{for} \ n < -1. \end{aligned}$$

We accomplished these computations using the fact that Γ_3 satisfies the Farrell-Jones Isomorphism Conjecture in lower algebraic K-theory [3]. One of the steps in this project was to classify up to isomorphism the family $\mathcal{VC}$ of all virtually cyclic subgroups of Γ_3 (see [6, Section 3]), and it is here where the problem arises. The list of groups appearing in [6, Lemma 3.4] is *incomplete*.

Let us recall that any virtually infinite cyclic group is either of type $F \rtimes_\alpha \mathbb{Z}$, where F is a finite group, or it maps onto D_∞ with finite kernel (see [6, Lemma 3.2]). It is well known that the group $F \rtimes_\alpha \mathbb{Z}$ can occur in Γ_3 only if $C_{\Gamma_3}(F)$ contains an element of infinite order (see [6, p. 340]).

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In [6, p. 341], we claimed that $C_{\Gamma_3}(\langle S_1, S_2\rangle) \cong C_{\Gamma_3}(D_2) = \langle S_4\rangle \cong \mathbb{Z}/2$, $C_{\Gamma_3}(\langle S_1, S_3\rangle) \cong C_{\Gamma_3}(D_3) = \langle S_1, S_3, S_4S_3S_4\rangle \cong (\mathbb{Z}/2)^3$, but these computation are *erroneous* (here D_n denotes the dihedral group of order $2n$). R. Akhtar pointed out to the author that in fact these groups are *infinite*. The following are the correct computations of these groups (see [4]):

$$C_{\Gamma_3}(\langle S_1, S_2\rangle) \cong D_\infty \rtimes \mathbb{Z}/2, \quad \text{and} \quad C_{\Gamma_3}(\langle S_1, S_3\rangle) \cong D_\infty \rtimes D_2.$$

This fact alters the classification of the virtually infinite cyclic subgroups of Γ_3 given in [6, Lemma 3.4]. This problem is settled in [5, Section 3], where we classify up to conjugacy the maximal virtually infinite cyclic subgroups of Γ_3.

Appealing to this new information, we want to emphasize that the group $D_2 \times \mathbb{Z}$ is now part of the family $\mathcal{VC}$ of virtually cyclic subgroups of Γ_3, and that the *essential* problem arises here. It is well known that if $Q = D_2 \times \mathbb{Z}$, the relative assembly map appearing in [6, Theorems 2.2, and 4.7]) is *not* an isomorphism. Since $K_0(\mathbb{Z}Q)$ is infinitely generated (see [2,1]), the map $\mathcal{A}_{\mathcal{F}_Q}$ appearing in [6, Theorem 4.1]) *fails* to be an isomorphism for $Q = D_2 \times \mathbb{Z}$. Therefore the reduction to finite subgroups *fails*.

In order to complete the computations of the lower algebraic K-groups of the integral group ring of Γ_3, we must compute the homology groups

$$H_n^{\Gamma_3}(E_{\mathcal{VC}}(\Gamma_3); \mathbb{KZ}^{-\infty}), \quad \text{for} \quad n < 2.$$

These computations are carried out in [5, Sections 4 and 5]. The result of these computations can be summarized in the following theorem (see [5, Theorem 1.1]):

Theorem *Let* $\Gamma_3 = O^+(3,1) \cap GL(4,\mathbb{Z})$. *Then the lower algebraic K-theory of the integral group ring of* Γ_3 *is given as follows*:

$$Wh(\Gamma_3) \cong \bigoplus_\infty \mathbb{Z}/2$$
$$\tilde{K}_0(\mathbb{Z}\Gamma_3) \cong \mathbb{Z}/4 \oplus \mathbb{Z}/4 \oplus \bigoplus_\infty \mathbb{Z}/2$$
$$K_{-1}(\mathbb{Z}\Gamma_3) \cong \mathbb{Z} \oplus \mathbb{Z}, \ \textit{and}$$
$$K_n(\mathbb{Z}\Gamma_3) = 0 \ \textit{for} \ n < -1,$$

where the expression $\bigoplus_\infty \mathbb{Z}/2$ *refers to a countable infinite sum of* $\mathbb{Z}/2$.

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